QUIZ 7

Let D_n be the dihedral group of order $2n$.

(1) (**2 points**) Show that $\langle a \rangle$ is a normal subgroup of D_n (**bonus**: prove this by coming up with a concrete homomorphism from $D_n \to \mathbb{Z}_2$ which has kernel equal to this group).

(2) (**5 points**) Put the following elements of D_6 into the form $a^m b^\epsilon$ with $0 \leq m < 6$ and $0 \leq \epsilon < 2$.
   (a) a^4baba^4ba
   (b) aba^3baba^3
   (c) a^{10}
   (d) aba^{-1}

(3) (**3 points**)
   (a) Show that if $a^m = b^\epsilon$ for $0 \leq m < n$ and $0 \leq \epsilon < 2$ then m and ϵ are equal to zero (**hint** go back to the geometry of how a and b were defined. Your answer should start by stating that "a^m is a —- but b^ϵ is a —- if $\epsilon \neq 0$" and then work out why this means $m = 0$ and $\epsilon = 0$).
   (b) In class we showed that every element of D_n can be written as $a^m b^\epsilon$ with $0 \leq m < n$ and $0 \leq \epsilon < 2$. Show this form is unique. (Your first line should read "if $a^m b^\epsilon = a^{m'} b^{\epsilon'}$" and your last line should read "and this is impossible by (3a)").