

## HOMEWORK 5

Prove the following.

- (1) Research or ask about the Quaternion group of order 8. Show explicitly that it is not isomorphic to  $D_4$  (hint, count the number of elements of order 2 in both groups).
- (2) Explicitly express  $D_4$  as a subgroup of  $S_4$  by following what  $D_4$  does to the vertices of a square.
- (3) Look up what the center of a group is in your textbook. Show that  $D_n$  has a non-trivial center if and only if  $n$  is even. (Hint: it may be helpful to first prove that  $2x = 0 \pmod n$  has a solution if and only if  $n$  is even.)
- (4) Write a subgroup of  $GL(2, \mathbb{R})$  which is isomorphic to  $D_4$  (hint: write the matrix which corresponds to the linear maps of  $\mathbb{R}^2$  preserving the square).