

## HOMEWORK 2

- (1) Write down an associative and a non-associative multiplication table with entries in  $\{a, b, c\}$  (prove they are associative or not, but do you really want to do 27 calculations?).
- (2) Let  $G := \mathbb{R} - \{-1\}$  and define  $a * b := a + b + a \cdot b$  (here  $+$  and  $\cdot$  have their usual meanings). Show that  $(G, *)$  is a group.
- (3) Let  $\mathbb{H} := \mathbb{R}^4$  (which we think of as  $\mathbb{R} \times \mathbb{R}^3$ ). Use usual facts about dot and cross products from calc III to show that  $\mathbb{H}$  equipped with the binary operation

$$(a, \vec{v}) * (b, \vec{w}) := (ab - \vec{v} \cdot \vec{w}, a\vec{v} + b\vec{w} + \vec{v} \times \vec{w})$$

is associative and has an identity (here  $\cdot$  and  $\times$  denote the usual dot product and cross products in  $\mathbb{R}^3$ ).

- (4) Let  $g$  be an element of a group. Show that  $(g^{-1})^{-1} = g$ .