

HOMEWORK 2

- (1) Write down an associative and a non-associative multiplication table with entries in $\{a, b, c\}$ (prove they are associative or not, but do you really want to do 27 calculations?).
- (2) Let $G := \mathbb{R} - \{-1\}$ and define $a * b := a + b + a \cdot b$ (here $+$ and \cdot has their usual meanings). Show that $(G, *)$ is a group.
- (3) Let $\mathbb{H} := \mathbb{R}^4$ (which we think of as $\mathbb{R} \times \mathbb{R}^3$). Use usual facts about dot and cross products from calc III to show that \mathbb{H} equipped with the binary operation

$$(a, \vec{v}) * (b, \vec{w}) := (ab - \vec{v} \cdot \vec{w}, a\vec{v} + b\vec{w} + \vec{v} \times \vec{w})$$

is associative and has an identity (here \cdot and \times denote the usual dot product and cross products in \mathbb{R}^3).

- (4) Let g be an element of a group. Show that $(g^{-1})^{-1} = g$.