HOMEWORK 1

(1) Perform the following operations.

(a) $(1+i)(3+i+k)$
(b) $(2 \mod 11)(8 \mod 11)$
(c) $(a+bi+cj+dk)(a-bi-cj-dk)$
(d) $(1+i+k)^{-1}$
(e) $7^{-1} \mod 11$
(f) $-5 \mod 13$.
(g) $(1002)(1000) \mod 1003$.[1]
(h)

$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

(i)

$$\begin{pmatrix} 1 \mod 7 & 2 \mod 7 \\ 3 \mod 7 & 2 \mod 7 \end{pmatrix}^{-1}$$

(j)

$$\begin{pmatrix} i & i+j \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & k \\ k & 1 \end{pmatrix}$$

(2) Let

$$\omega := \frac{(1+i+j+k)}{2}$$

.

(a) Show that $\omega \in \mathbb{S}^3$.
(b) Compute the multiplicative order of ω.
(c) Find ω^{-1}.

(3) Let $q \in \mathbb{H}$. Show that $q \in \mathbb{S}^3$ if and only if $q^{-1} = \overline{q}$.

(4) A quaternion is called *real* if and only if it equals its own conjugate (i.e., the given quaternion's i, j and k components are each 0.) Show that a quaternion q commutes with every other quaternion if and only if q is real (**Hint:** Take this slowly! First, figure out what it means for q to commute with i, then figure out what it means for it to commute with j.)

[1] use a trick!