Let $x = (1\,2\,3\,4)$, $y = (1\,2)$ (Note the relations: $x^4 = e, y^2 = e, xyx = yx^{-1}y$). Then

$$
\begin{aligned}
(1) &= (x^4 = y^2) \\
(1\,2) &= y \\
(1\,3) &= yxyx^{-1}y = x^{-1}yx^2y \\
(1\,4) &= x^{-1}yx = (yxyx^2) \\
(2\,3) &= xyx^{-1} = (xyx^3) \\
(2\,4) &= xyx^2y = (yx^2yx^{-1} = xyxyx^{-1}yx^{-1}) \\
(3\,4) &= x^2yx^2 \\
(1\,2\,3) &= x^{-1}yx^2 = yxyx^{-1} = (yx^2yxy) \\
(1\,2\,4) &= x^2yx^{-1} \\
(1\,3\,2) &= x^2yx = xyx^{-1}y \\
(1\,3\,4) &= xy \\
(1\,4\,2) &= xyx^2 = yx^{-1}yx
\end{aligned}
\qquad
\begin{aligned}
(1\,4\,3) &= yx^{-1} = (yx^3 = xyxy) \\
(2\,3\,4) &= yx \\
(2\,4\,3) &= x^{-1}y = (x^3y) \\
(1\,2\,3\,4) &= x \\
(1\,2\,4\,3) &= xyx = yx^{-1}y \\
(1\,3\,2\,4) &= yx^2 \\
(1\,3\,4\,2) &= yxy = x^{-1}yx^{-1} \\
(1\,4\,2\,3) &= x^2y \\
(1\,4\,3\,2) &= x^{-1} = (x^3) \\
(1\,2)(3\,4) &= x^2yx^2y = xyx^{-1}yxy \\
(1\,3)(2\,4) &= x^2 \\
(1\,4)(2\,3) &= yx^2y = (xyx^2yx)
\end{aligned}
$$

The *length* of a word

(1) $$w = x_1^{i_1} \cdots x_n^{i_n}$$

is defined as

$$\sum_{j=1}^{n} |i_j|.$$

(this depends on the word, not the element!) That it is equal to the number of letters it takes to write $w$ (negative powers still count for positive letters). For instance the word $x^2yx^{-1}$ has length 4. In the table all words of minimal length which represent a given element of $S_4$ are written without parenthesis. Other words are written with parenthesis.

- This table proves that $S_4$ is generated by $(1\,2)$ and $(1\,2\,3\,4)$ since we can see from inspection that every element of $S_4$ is a word in $x$ and $y$ (**excer**: prove that in general $S_n$ is generated by $(1\,2)$ and $(1 \cdots n)$ .... clearly making this kind of table is not the best approach).
- Notice that many elements must be written as words which switch back and forth between $x$'s and $y$'s at least twice. This surprises many students. More formally, by combining repeats, we can assume that word appearing in equation(1) above satisfies $x_i \neq x_{i+1}$ (so $x^2yxx^2$ would not qualify but $x^2yx^3$ would) then we define the *chunk length* of $w$ to be $n$. One of the most common mistakes students make is to assume that the chunk length cannot be greater than 2. However in this table we see several elements, e.g. $(1\,3)$ and $(2\,4)$, who do not have a word representing them of chunk length less than 4.
- The other remarkable thing to notice about this table is that there isn't much else remarkable about this table. For most groups and for most generating sets there isn't a "sexy' ' way of figuring out the best way to represent a given element as a word in those generators.