Answers to Exam 1

$\vec{u} = \langle 1, 0, -2 \rangle$

$\vec{w} = \langle 1, 2, 3 \rangle$

a) $\vec{u} - 2\vec{w} = \langle -1, -4, -8 \rangle$

b) $\vec{u} \cdot \vec{w} = 1 - 6 = -5.$

c)
$$\left|\begin{array}{ccc|cc} i & j & k & i & j \\ 1 & 0 & -2 & 1 & 0 \\ 1 & 2 & 3 & 1 & 2 \end{array}\right.$$

$$= (0\hat{i} - 2\hat{j} + 2k) - (3j - 4\hat{i} + 0k)$$

$$= 4\hat{i} - 5\hat{j} + 2k$$

d) $\sqrt{\vec{u} \cdot \vec{u}} = \sqrt{1^2 + 0 + (-2)^2} = \sqrt{1+4} = \sqrt{5}$

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2) $A = (1,0,1)$ $B = (3,-1,2)$ $C = (2,3,0)$

$\vec{u} = \overrightarrow{AC} = \langle 1, 3, -1 \rangle$ 5

$\vec{v} = \overrightarrow{AB} = \langle 2, -1, 1 \rangle$

$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ 1 & 3 & -1 \\ 2 & -1 & 1 \end{vmatrix}$$

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$(3-1)\,i - (1+2)\,j + (-1-6)\,k$

$= 2i - 3j - 7k.$

$(\vec{u} \times \vec{v}) \cdot \langle x, y, z \rangle = d$

$\Leftrightarrow 2x - 3y - 7z = d$

Now solve for d, we use A

$2 - 7 = -5 \Rightarrow d = 5$ & equation

$= \boxed{2x - 3y - 7z = -5}$ 5

3) We want to find

$$\int_0^{\pi} \|\vec{r}\,'(t)\|\, dt.$$

Let's first calculate $\|\vec{r}\,'(t)\|$.

$$\vec{r}\,'(t) = \langle 3\cos t, -5\sin t, 4\cos t \rangle$$

$$\|\vec{r}\,'(t)\| = \sqrt{9\cos^2 t + 25\sin^2 t + 16\cos^2 t}$$

$$= \sqrt{25\cos^2 t + 25\sin^2 t}$$

$$= \sqrt{25\,(\cos^2 t + \sin^2 t)}$$

$$= \sqrt{25}$$

$$= 5.$$

Thus $\displaystyle\int_0^{\pi} \|\vec{r}\,'(t)\|\, dt = \int_0^{\pi} 5\, dt = \boxed{5\pi}$

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4)

$\vec{r}'(t) = \cos t\, \hat{i} + e^t \hat{j} + 2t\, k$ 5

$\vec{r}''(t) = -\sin t\, \hat{i} + e^t \hat{j} + 2k$ 5

$\vec{r}'(t) \times \vec{r}''(t) =$

$$\begin{vmatrix} \hat{i} & \hat{j} & k \\ \cos t & e^t & 2t \\ -\sin t & e^t & 2 \end{vmatrix}$$

$= (2e^t - 2te^t)\,\hat{i} - (2\cos t + 2t\sin t)\,\hat{j}$

$+ (e^t \cos t + e^t \sin t)\, k.$

eval @ $t=0$

$(2\hat{i} - 2\hat{j} + k)$

$\|\vec{r}'(0) \times \vec{r}''(0)\| = \sqrt{4+4+1} = \sqrt{9} = 3.$

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$$\|r'(0)\| = \|i + j\| = \sqrt{2}$$

$$k(0) = \frac{\|r'(0) \times r''(0)\|}{\|r'(0)\|^3} = \frac{3}{2^{3/2}} = \frac{3}{2\sqrt{2}} = \boxed{\frac{3\sqrt{2}}{4}}$$

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5) $f(x,y) = e^{xy}\sin(y^3+1)$

$$\frac{\partial f}{\partial x} = y e^{xy}\sin(y^3+1)$$ 5

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$$\frac{\partial^2 f}{\partial y\,\partial x} = \frac{\partial^2 f}{\partial x\,\partial y} = \boxed{e^{xy}\sin(y^3+1) + xye^{xy}\sin(y^3+1) + ye^{xy}\cos(y^3+1)(3y^2)}$$

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$$\frac{\partial^2 f}{\partial x^2} = \boxed{y^2 e^{xy}\sin(y^3+1)},$$

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