Answers to Quiz 3

Mark O'Brien

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1. (Q) Compute

$$\frac{\partial^2}{\partial x \, \partial y}\left(\frac{xy}{x^2+y^2}\right).$$

(A) This is straight forward.

$$\begin{aligned}
\frac{\partial}{\partial y}\left(\frac{xy}{x^2+y^2}\right) &= \frac{x\,(x^2+y^2) - xy\,(2y)}{(x^2+y^2)^2} \\
&= \frac{(x^3 + x\,y^2) - 2xy^2}{(x^2+y^2)^2} \\
&= \frac{(x^3 - x\,y^2)}{(x^2+y^2)^2}
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial}{\partial x}\left(\frac{(x^3 - x\,y^2)}{(x^2+y^2)^2}\right) &= \frac{(3x^2 - y^2)(x^2+y^2)^2 - (x^3 - x\,y^2)(4\,x)\,(x^2+y^2)}{(x^2+y^2)^4} \\
&= \frac{(3x^2 - y^2)(x^2+y^2) - (x^3 - x\,y^2)(4\,x)}{(x^2+y^2)^3} \\
&= \frac{6x^2y^2 - x^4 - y^4}{(x^2+y^2)^3}
\end{aligned}$$

2. (Q) Show that the the following function does not have a limit as (x, y) goes to $(0, 0)$

$$\frac{x^2}{x^2+y^2}.$$

(A) If you go along the curve $y = x$ you get a limit equal to $1/2$ if you go along the curve $x = 0$ you get 0. Actually with this curve any two straight lines give a different answer.

3. **Bonus**

(Q) Prove the following: if $\overrightarrow{r}(t)$ is a curve with $\|\overrightarrow{r}(t)\|$ constant, then $\overrightarrow{r}(t)$ and $\frac{d}{dt}\overrightarrow{r}(t)$ are perpendicular.

(A) We need to show that $\frac{d}{dt}\left(\overrightarrow{r}(t)\right) \cdot \overrightarrow{r}(t) = 0$. We are given that $\|\overrightarrow{r}(t)\|$ is constant which means that $\overrightarrow{r}(t) \ \cdot \overrightarrow{r}(t) \ = c$ where c is some constant (positive) real number (equal to the length squared). Then using the product rule we get

$$\begin{aligned}
0 = \frac{d}{dt}c &= \frac{d}{dt}\left(\overrightarrow{r}(t) \cdot \overrightarrow{r}(t)\right) \\
&= \frac{d}{dt}\left(\overrightarrow{r}(t)\right) \cdot \overrightarrow{r}(t) + \overrightarrow{r}(t) \cdot \frac{d}{dt}\left(\overrightarrow{r}(t)\right) \\
&= \frac{d}{dt}\left(\overrightarrow{r}(t)\right) \cdot \overrightarrow{r}(t) + \frac{d}{dt}\left(\overrightarrow{r}(t)\right) \cdot \overrightarrow{r}(t) \\
&= 2\frac{d}{dt}\left(\overrightarrow{r}(t)\right) \cdot \overrightarrow{r}(t)
\end{aligned}$$

That is $\frac{d}{dt}\left(\overrightarrow{r}(t)\right) \cdot \overrightarrow{r}(t) = 0$ which is what we wanted.