Math 180 – Spring 2011 Name: _____

Final Exam

Write your solutions in an exam booklet. Nothing written on this sheet will be graded.

(20 pts) **1.** Evaluate the following limits, or show that they do not exist.

(a) $\lim_{h\to 0} \dfrac{e^{x+h} - e^x}{h}$

(b) $\lim_{x\to 0} \dfrac{|x^2 - 4|}{3x + 6}$

(30 pts) **2.** Differentiate the following functions. Leave your answers in unsimplified form so it is clear what method you used.

(a) $\dfrac{x}{2 + \sin(x)}$

(b) $\arctan(e^x)$

(c) $\ln(e^x + 1)$

(20 pts) **3.** Calculate the indefinite integrals.

(a) $\displaystyle\int \frac{e^{-\frac{1}{x}}\, dx}{x^2}$

(b) $\displaystyle\int x \sin(x^2)\cos(x^2) dx$

(20 pts) **4.** Calculate the definite integrals.

(a) $\displaystyle\int_0^{\pi/4} \sec^2(x) dx$

(b) $\displaystyle\int_0^2 \frac{x}{(x^2+2)^2} dx$

(20 pts) **5.** Let $F(x) = \displaystyle\int_1^{x^2} \ln(t)\, dt$.

(a) Compute $F(-1)$.

(b) Find the derivative $F'(x)$.

The exam CONTINUES on the next page.

(30 pts) **6.** Consider the function $f(x) = \frac{\ln(x)}{x}$ defined for $x > 0$.

(a) Find the critical point(s) of f.

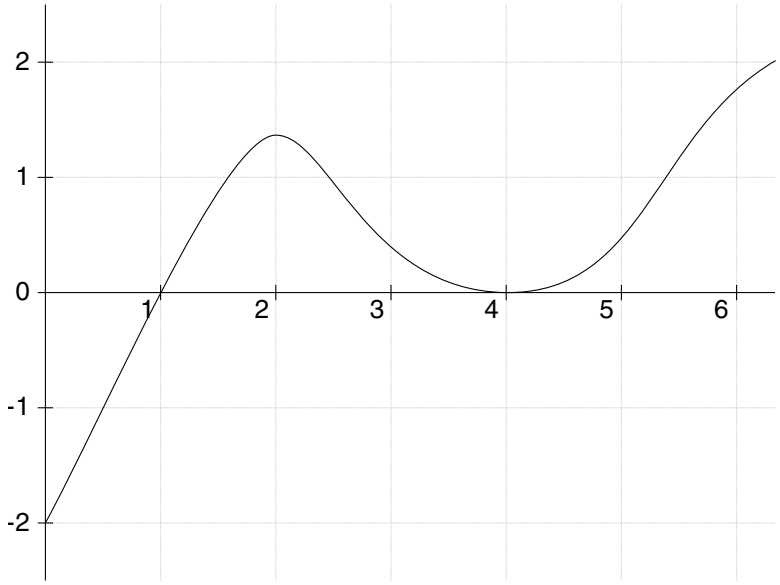
(b) Classify each critical point as a local minimimum, local maximum, or neither. Justify your answers.

(c) Find the absolute minimum and absolute maximum of f on the interval $[1, e^2]$.

(30 pts) **7.** Shown below is the graph of $f'(x)$, the **derivative** of the function $f(x)$.

(a) Using the graph of $f'(x)$ below, determine the intervals where f is increasing, decreasing, concave up, and concave down.

(b) Given that $f(0) = 1$, sketch the graph of the function $f(x)$. On your graph, clearly label all maxima, minima, and inflection points.



This is the graph of the **derivative** of $f(x)$.

(20 pts) **8.** Consider the curve defined by the equation $x^3 + y^3 = x^2 + y^2$.

(a) Use implicit differentiation to find the derivative $\dfrac{dy}{dx}$ in terms of x and y.

(b) Find an equation for the tangent line to this curve at the point $(\frac{5}{9}, \frac{10}{9})$.

(10 pts) **9.** A function g is defined on the interval $[1, 3]$ by $g(x) = x^4 - x + 1$. Let $h(x) = g^{-1}(x)$ be the inverse function. Compute $h'(15)$.

Hand in this sheet along with your exam booklet!