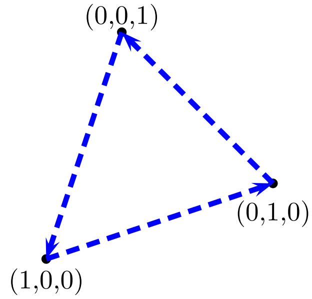
Answer the questions in the exam booklet. Answers provided on this sheet will be ignored.

1. (15 points) For this problem let
$$\overrightarrow{F}(x, y, z) = z\widehat{k}.$$
   (a) (5 points) By calculating the curl or explicitly finding a function, verify that $\overrightarrow{F}$ is the gradient of some function.
   (b) (5 points) Directly compute the line integral
$$\int_{\mathcal{C}} \overrightarrow{F} \bullet d\overrightarrow{s}$$
   where $\mathcal{C}$ is the boundary curve of a triangle, $\mathcal{W}$, with vertices $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$ transversed in that order.



   (c) (5 points) Explain how we could have known the answer to the previous part without doing a calculation.

2. (15 points) Let $\mathcal{W}$ be the same triangle from the previous problem but for this problem we instead let
$$\overrightarrow{F}(x, y, z) = x\widehat{k}.$$
Verify Stokes theorem for this vector field and this surface.

3. (30 points) (a) (5 points) Parametrize a cone of radius R and height H which is situated as on the board.
   (b) (5 points) Compute a normal vector to this cone which points outwards.
   (c) (5 points) Show that the surface area of this cone (not including the top of the cone) is
$$\pi R\sqrt{R^2 + H^2}.$$
   (d) (5 points) Compute the suface integral
$$\iint_{\text{Cone}} \overrightarrow{F} \bullet \mathrm{d}\overrightarrow{S}$$
   over this cone where
$$\overrightarrow{F}(x, y, z) = x\widehat{i}.$$
   (e) (5 points) Compute the volume enclosed by this cone by using the previous part and the divergence theorem. Please be clear how exactly you are using the divergence theorem. For instance why can you ignore the top of the cone?
   (f) (5 points) Compute the volume instead by doing a triple integral.

4. (20 points) Find the volume contained inside of an ellipsoid

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$$

by doing a triple integral and changing coordinates via

$$\Phi(x, y, z) = (ax, by, cz).$$

5. (20 points) Let $\mathcal{W}$ be the portion of the unit sphere in the first octant. In other words, we are considering the region

$$x^2 + y^2 + z^2 \leq 1$$
$$x \geq 0$$
$$y \geq 0$$
$$z \geq 0.$$

Compute the integral

$$\iiint_{\mathcal{W}} xyz \; dxdydz$$

in your favorite two coordinate systems (two of cartesian, cylindrical, or spherical).

6. (50 points) **Bonus!!**

Let ω be the 1-form

$$\omega = yz\,dx + xz\,dy + xy\,dz$$

compute $d\omega$.