# Exam 2 Solutions

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1. (a)

$$\frac{d}{dx}(x^2\sin(x)) = 2x\sin(x) + x^2\cos(x).$$

(b)

$$\frac{d}{dx}(x^{\frac{1}{4}} - x^{-\frac{2}{3}}) = \frac{1}{4}x^{-\frac{3}{4}} + \frac{2}{3}x^{-\frac{1}{3}}$$

(c)

$$\frac{d}{dx}e^{-(a+x^2)} = -2xe^{-(a+x^2)}$$

2. (a) Differentiating both sides of the equation gives

$$6x - 2x\frac{dy}{dx} - 2y + 2y\frac{dy}{dx} = 0$$
$$\Rightarrow \frac{dy}{dx}|_{(1,2)} = \frac{y-3x}{y-x}|_{(1,2)} = \frac{2-3}{2-1} = -1.$$

(b) From the first part, we already know that the tangent line has slope -1. So we only need to find the $y$ intercept. Using $y = -x + b$ and plugging in the point $(1, 2)$ we get $2 = -1 + b$ so $b = 3$. So the line is $y = -x + 3$.

3. (a)

$$f(x) = x^3 + 3x^2 - 9x - 10$$
$$\Rightarrow f'(x) = 3x^2 + 6x - 9.$$

and

$$f'(x) = 0 \Leftrightarrow 3x^2 + 6x - 9 = 0$$
$$\Leftrightarrow x^2 + 2x - 3 = 0$$
$$\Leftrightarrow (x+3)(x-1) = 0$$
$$\Leftrightarrow x = -3 \text{ or } x = 1.$$

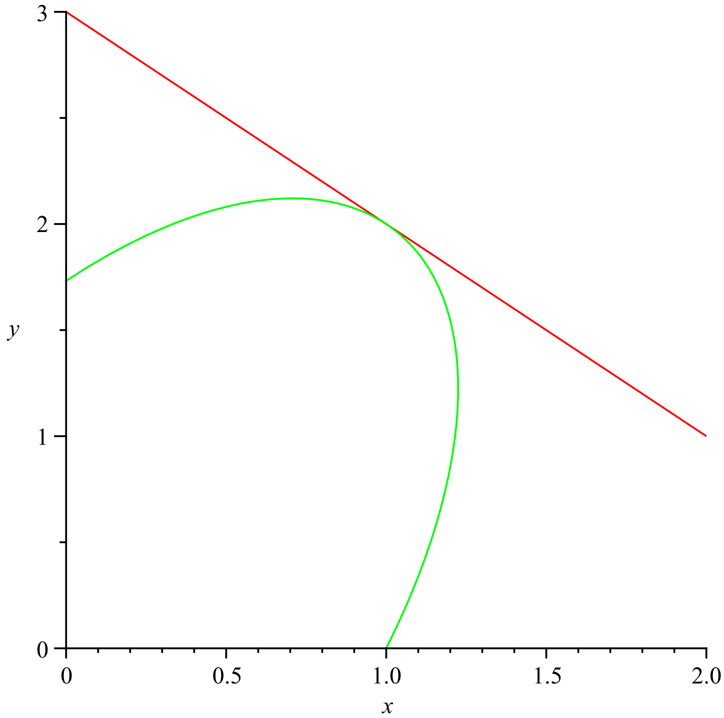


Figure 1: Image showing the solution to number 2.

(b) $f$ is decreasing only when $f'(x) < 0$. This happens only when $-3 < x < 1$.

(c) The only points that we need to check are $x = -3$ and $x = 2$ (why could be ignore $x = -4$ and $x = 1$?). But since $f(-3) = 17$ and $f(2) = -8$ we can conclude that $f(-3) = 17$ is the maximum of the function on the given interval.

4. This is a L'Hopital's rule problem.

$$\begin{aligned} &\lim_{x\to 0} \frac{\ln(1+x)}{x^2 - x} && (\text{form is } \frac{0}{0}) \\ &\overset{H}{=} \lim_{x\to 0} \frac{\frac{1}{1+x}}{2x-1} \\ &= \frac{1}{-1} \\ &= -1. \end{aligned}$$

5. The relevant function is given by pythagorean theorem, namely

$$D = (x-3)^2 + y^2. \tag{1}$$

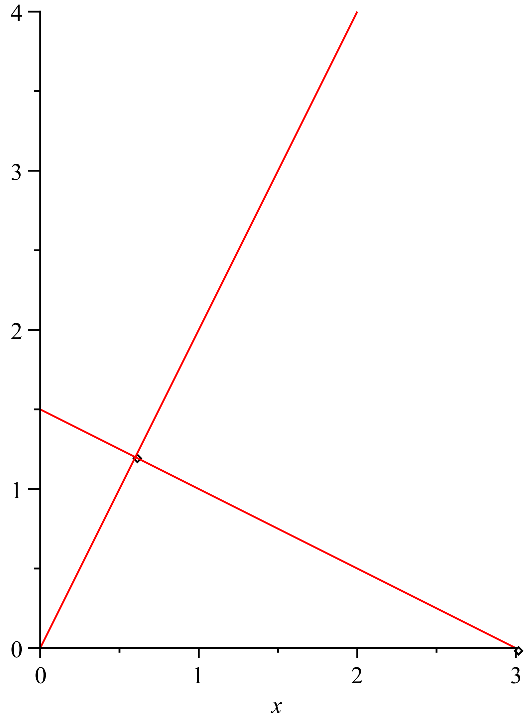


Figure 2: Image showing the solution.

(note we can use the square of the distance function instead of the distance function itself since if we minimize one we minimize the other). The constraint is given by the defining line, $y = 2\,x$. Putting this into equation (2) gives

$$D(x) = (x-3)^2 + (2x)^2. \tag{2}$$

Next, as usual, we take a derivative and set it equal to 0.

$$\begin{aligned} D'(x) = 2(x-3) + 8x &= 0 \\ \Leftrightarrow 10x &= 6 \\ \Leftrightarrow x &= \frac{3}{5}. \end{aligned}$$

Note that since $D''(x) = 10 > 0$ the second derivative test says that this point that we found is actually minimizes the function $D$. Lastly we use the original constraint, $y = 2x$ to get $y = 2(3/5) = 6/5$. So the final answer is

$$\left(\frac{3}{5}, \frac{6}{5}\right).$$

6. No solution provided.