

EXAM 1 MATERIAL

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Here are the materials for exam 1 which will be given Friday July 15, 2011 during class.

- (1) Vectors
 - (a) **Basic Operations**
 - Converting between $a\hat{i} + b\hat{j} + c\hat{k}$ notation and (a, b, c) notation.
 - Addition.
 - Scalar multiplication.
 - Dot Product.
 - Cross Product. (Both methods!)
 - (b) **Geometric Significance** You should know how to interpret all of these operations geometrically. For instance
 - Given a pair of vectors find the length of the diagonal of a parallelogram which they span.
 - Given vectors you should be able to tell whether or not they are orthogonal.
 - Given a pair of vectors you should be able to find a third perpendicular to both.
 - Given a pair of vectors find the area of the triangle/parallelogram which they span.
- (2) Curves. Be able to calculate a curve's
 - (a) **Velocity**
 - (b) **Speed**
 - (c) **Length**
 - Calculate a curve's length given a start and end time.
 - Calculate its arc-length parameterization.
- (3) Given a function

$$F: \mathbb{R}^n \rightarrow \mathbb{R}^m \tag{1}$$

- (a) **Notation**
 - What does the notation in (1) mean? (This is one of two things on the exam that is **not** covered in the text!)
 - What do we call a function with $n = 1$, etc.
 - Match a function to its graph.
 - (b) **Limits**
 - Show that a limit does **not** exist.
 - Be able to state the formal definition of a limit and continuity.
- (4) Differential Calculus of multi-variable functions.
 - (a) **Basics**
 - What was the definition of a partial derivative given in class?
 - Calculate the partial derivatives of a function.

- Verify that the chain rule holds for a pair of general composable functions. (This could be slightly more general than what the book deals with).
 - Calculate the tangent plane of a surface $F(x, y, z) = 0$ at a given point.
- (b) **Optimization**
- Find and classify the critical points of a function $f: \mathbb{R}^2 \rightarrow \mathbb{R}$.
 - Explain what the point of Lagrangian multipliers is. For instance given a list of problems explain whether or not you would use the method of Lagrangian multipliers or the method from the previous bullet.
 - Apply the Lagrangian multiplier method to a specific problem.
- (5) Bonus material. (These are the generalizations of concepts learned in class that weren't asked on the homework or covered in the text.) For instance: be able to
- (a) Calculate the sign of a permutation.
 - (b) Calculate the determinant of a 4 by 4 (or larger!) matrix.
 - (c) Find a local maximum or minimum of a function $\mathbb{R}^3 \rightarrow \mathbb{R}$.